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functional potentiometers, with which a representation of functions of one independent variable can be obtained. However, such potentiometers are useful only for the representation of any one function of one independent variable and moreover for a limited class of these. The production of the potentiometers with shaped mandrels used up to this time is expensive and their accuracy is none too high. In 1946, the author proposed and developed a new method for representation of functions using electrical profiling (curve-fitting) of a potentiometer by means of forced distribution of electric voltage along the potentiometer according to a given law (Certificate of Authorship No 72856 awarded on 19 April 1947). This method of electrical representation of functions covers a broad class of both monotonically and nonmonotonically varying functions and sign-variable functions.

A very important characteristic of the new method is that the law governing the change of voltage can be set and changed arbitrarily for the same potentiometer, which is especially important for integrating units, for example, where the integrand is established according to any required law. Moreover, the new method is useful not only for the representation of functions of one variable but also for functions of several independent variables. In addition, this method provides much higher accuracy in the operation of the device, i.e., of the order of hundredths of a percent. The substance of the new method is that the resistance of a linear potentiometer is divided into a number of sections which are shunted by auxiliary resistors with the purpose of forced distribution of voltage according to a given law. The value and number of the resistors are determined by the given law of voltage variation and the accuracy required.

A potentiometer circuit with shunting resistors is shown in Figure 1. The voltage taken off between point 1 (the beginning) and the sliding contact of the potentiometer changes according to the law

$$U = U_0 f(x), \quad (1)$$

where  $f(x)$  is the given function;  $x$  is the relative displacement of the sliding contact of the potentiometer (proportional to the independent variable); and  $U_0$  is the voltage applied to the potentiometer. Here the linear potentiometer with a continuous winding along its length is divided into a number of sections, the resistances of which we designate  $a_1, a_2, \dots, a_n$ . These sections are shunted by the resistances  $b_1, b_2, \dots, b_n$ . In addition, the additional resistances  $r_0$  and  $r_{n+1}$  are connected in series with the potentiometer to set on the potentiometer the values of the resistances corresponding to the initial and final values of the function. With the help of the shunting resistances  $b_1, b_2, \dots, b_n$ , the voltage distribution along the length of the linear potentiometer is varied to correspond with the given functional law. Actually, the resistance of any  $k$ th section of the functional potentiometer together with the shunting resistance is

$$r_k = \frac{a_k b_k}{a_k + b_k}. \quad (2)$$

The resistance of the functional potentiometer from the beginning (point 1) to the end of the  $k$ th section is

$$R_k = r_0 + \sum_{k=1}^n \frac{a_k b_k}{a_k + b_k}. \quad (3)$$

The voltages distributed along the length of the potentiometer are proportional to the resistances

$$\frac{U_k}{U_0} = \frac{R_k}{R_0}, \quad (4)$$

where  $R_0$  is the total resistance of the entire functional potentiometer together with the additional and shunting resistances, i.e.,

$$R_0 = r_0 + r_{n+1} + \sum_{k=1}^n r_k. \quad (5)$$

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If the voltage distribution along the length of the potentiometer must be defined by formula (1), it is obvious that the values of the resistances  $r_0, r_1, r_2, \dots, r_n, r_{n+1}$  must be:

$$\begin{aligned} r_0 &= R_0 f(x_0), \\ r_1 &= R_0 [f(x_1) - f(x_0)], \\ r_n &= R_0 [f(x_n) - f(x_{n-1})], \\ r_{n+1} &= R_0 - f(x_n) R_0; \end{aligned} \quad (6)$$

where  $x_0, x_1, \dots, x_n$  are the relative values of the quantity  $x$  at the points where the sections are shunted. Here we assume that the resistance  $R_0$  corresponds to some maximum value of the function  $f(x)$ , which we take as unity, so that  $f(x_{\max}) = 1$ . Therefore, in the calculation of the resistances  $r_0, r_1, r_2, \dots$  according to formula (6), we must substitute therein the relative values of the function  $f(x)$ .

By determining from formula (6) the values of the resistances  $r_0, r_1, \dots, r_n$  of the functional potentiometer and substituting them in (2), we obtain the values of shunting resistances  $b_1, b_2, \dots$  required in order that the voltage will vary along the length of the potentiometer according to the given functional law. The potentiometer is divided into a number of sections such that the dependency  $f(x)$  will be linear within one section with the required degree of accuracy. Thus, the number of sections and the shunting points on the potentiometer can be determined by a linear approximation of the curve  $f(x)$  by straight-line segments. This will give the required degree of accuracy.

For any intermediate point  $x$  lying between two shunting points  $x_1$  and  $x_2$ , the voltage taken from the potentiometer will have some relative error because the resistance of the potentiometer section  $a_1$  is a linear voltage divider in the interval between  $x_1$  and  $x_2$ . Consequently, the curve of the function  $y = f(x)$  has a linear approximation in this interval (Figure 2).

The relative error of the linear approximation can be determined directly from consideration of Figure 2, namely:

$$\delta = \frac{\Delta U}{U_0} = 1 - \frac{U_1}{U_0} - \frac{U_2 - U_1}{U_0} \cdot \frac{x - x_0}{x_1 - x_0}, \quad (7)$$

where  $U_0$  is the voltage applied to the potentiometer and  $U_1$  and  $U_2$  are the potentiometer voltages corresponding to the shunting points at distances  $x_1$  and  $x_2$  from the beginning of the potentiometer (in relative units).

Thus, the voltage  $U$  taken from the potentiometer will vary continuously and smoothly according to the given functional law with a certain error  $\delta$  (the approximation error).

#### Potentiometers for Nonmonotonically Varying and Sign-Variable Functions

Voltages may also be distributed along the length of a potentiometer according to a given law by connecting additional resistances  $b_1, b_2, \dots, b_n$  between the supply source terminals and the shunting points  $x_0, x_1, \dots, x_n$  of the linear potentiometer (Figure 3). The sections are shunted from both terminals of the supply source, so that the additional resistances  $b_1, b_2, \dots, b_n$  may be considered as shunting resistances in the closed circuit of the supply source. By varying the value of these additional resistances, we can arbitrarily change the voltage distribution along the length of the potentiometer.

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Figure 4a shows the representation of the nonmonotonically varying function given in Figure 4b. The voltages proportional to the ordinates  $b_0$ ,  $b_{01}$ ,  $b_{02}$ , and  $b_{03}$  are given by the corresponding additional resistances while the shunting resistances  $b_1$ ,  $b_2$ , and  $b_3$  serve for electrical profiling (curve-fitting) of the linear potentiometer in the interval between  $b_0$  and  $b_{01}$ , then  $b_{01}$  and  $b_{02}$ , etc. If we are given a sign-variable function (Figure 5), the middle sections of the potentiometer are connected together as equipotential and at these points the voltage taken from the potentiometer is zero, since the point 0 serves as the beginning of the reading. The additional resistances  $b_0$ ,  $b_1$ , and  $b_2$  are used to obtain the required voltages at the points where they are connected.

The value of the additional resistance for any section of the potentiometer can be determined from the formula:

$$b_k = \frac{U_k}{\frac{U_{k+1} - U_k}{a_{k+1}} - \frac{U_k - U_{k-1}}{a_k}}, \quad (8)$$

where  $U_k$ ,  $U_{k-1}$ , and  $U_{k+1}$  are the voltages corresponding to the points  $x_k$ ,  $x_{k-1}$ , and  $x_{k+1}$  where the additional resistances are connected and  $a_k$  and  $a_{k+1}$  are the resistances of the sections of the linear potentiometer between the points  $x_{k-1}$ ,  $x_k$  and  $x_k$ ,  $x_{k+1}$ . If  $b_k$  turns out to be negative when calculated, it means that an additional resistance should be connected to the other terminal (minus) of the supply source.

#### Representation of Functions of Two Variables (Electric Conoid)

Many functions with which we must deal in computers cannot be expressed analytically and are given in the form of experimental tables or graphs, for example, ballistic functions. Up to this time, mechanical devices, the so-called conoids, have been used for representation of functions of two variables, since there have been no electrical methods of representing functions of two variables in the form of continuously varying voltages.

The author has proposed a method of representing functions in the form of an electric voltage using a functional potentiometer with variable shunting resistances. In this method, the principle of approximation of the given function by straight-line segments is used.

Suppose we are given a function  $U$  of two independent variables  $x$  and  $y$ .

We represent this function (Figure 6) in the form of an electric voltage  $U$  taken from a linear potentiometer  $P_x$ , the sliding contact of which is displaced in proportion to the value of the independent variable  $x$ . The sections of the linear potentiometer  $P_x$  are shunted by variable functional resistances  $b_1$ ,  $b_2$ , ...,  $b_n$ . The sliding contacts of these resistances rotate on a common shaft, the angle of deflection of which is proportional to the variable  $y$ . Two additional functional resistances  $r_0$  and  $r_{n+1}$ , the sliding contacts of which also rotate on the  $y$  shaft, are connected in series with the potentiometer  $P_x$ . They are used to set the initial and final values of the function.

To determine the shunting points  $x_0$ ,  $x_1$ ,  $x_2$ , ...,  $x_n$ , we make a number of graphs of the function  $U$  for some given values  $y_0$ ,  $y_1$ ,  $y_2$ , ...,  $y_n$  which cover the range from  $y = y_0$  to  $y = y_n$ , i.e., graphs of the functions

$$\begin{aligned} (U_x)_0 &= f(x, y_0), \\ (U_x)_1 &= f(x, y_1), \\ &\dots \\ (U_x)_n &= f(x, y_n). \end{aligned} \quad (9)$$

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Then we approximate each curve of (9) by straight-line segments in a manner that their approximation error does not exceed the assigned value. In approximating these curves, we select those values of the shunting points  $x_0, x_1, \dots, x_n$  which satisfy the assigned accuracy of approximation for all curves of (9).

From the conditions of the problem, the additional resistance  $r_0$  when the sliding contact of the potentiometer  $P_X$  is at point  $x_0$  must be (for any value of  $y$ ):

$$r_0 = f(x_0, y) R_0, \quad (10)$$

where  $R_0$  is the total given resistance of the functional potentiometer.

When the sliding contact of  $P_X$  is at the points  $x_1, x_2, \dots, x_n$ , the resistances  $r_1, r_2, \dots, r_n$  must be:

$$\begin{aligned} r_1 &= R_0 [f(x_1, y) - f(x_0, y)], \\ r_2 &= R_0 [f(x_2, y) - f(x_1, y)], \\ r_n &= R_0 [f(x_n, y) - f(x_{n-1}, y)] \\ \text{and} \quad r_{n+1} &= R_0 - f(x_n, y) R_0. \end{aligned} \quad (11)$$

Since we know the resistances  $r_0, r_1, r_2, \dots, r_n$  from formula (2), we can determine the value of the shunting resistances. For values of  $x$  between  $x_0$  and  $x_1, x_1$  and  $x_2, x_2$  and  $x_3$ , etc., the values of the function  $U = f(x, y)$  will be determined with a certain error, which will depend on the number of sections (the approximation error).

#### Representation of Functions of Three Independent Variables

The function

$$U = f(x, y, z) \quad (12)$$

of three independent variables  $x, y$ , and  $z$  cannot be represented by means of a mechanical conoid, since the conoid is a surface in a three-dimensional space. However, we can suggest an electrical method for representing the function (12) in the form of a voltage  $U$  dependent on three variables.

We take the potentiometer  $P_X$  (Figure 7), the sliding contact of which is displaced proportional to  $x$ . We assume that the additional and shunting resistances  $r_0, b_1, b_2, \dots, b_n, r_{n+1}$  of this potentiometer represent some functions of the two remaining variables  $y$  and  $z$ . We select the shunting points  $x_0, x_1, x_2, \dots, x_n$  of the potentiometer  $P_X$  in such a way that the error in the determination of  $U = f(x, \bar{y}, \bar{z})$ , where  $\bar{y}$  and  $\bar{z}$  are some discrete values of the parameters  $y$  and  $z$ , does not exceed the assigned value  $\delta$ . As we did in the case of a function of two variables, we made a number of graphs of the function  $U$  of the variable  $x$  for various discrete values  $\bar{y}$  and  $\bar{z}$  which cover the entire range of variation of the parameters  $y$  and  $z$ , i.e., the curves

$$U = f(x, \bar{y}, \bar{z}). \quad (13)$$

On the graphs of the curves (13), we select those values of  $x_0, x_1, \dots, x_n$ , between which the sections of the curves (13) can be approximated by straight lines with a sufficient degree of accuracy. We now determine the values of the resistances for the separate sections of the functional potentiometer which will reproduce the function (12).

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Obviously, for  $x = x_0$ , the additional resistance  $r_0$  should be:

$$r_0 = R_0 f(x_0, y, z) \quad (14)$$

for any values assigned to  $y$  and  $z$ .

In the same way, the resistances  $r_1, r_2, \dots, r_n$  of the functional potentiometer must be:

$$\begin{aligned} r_1 &= R_0 [f(x_1, y, z) - f(x_0, y, z)], \\ r_2 &= R_0 [f(x_2, y, z) - f(x_1, y, z)], \\ &\dots \\ r_n &= R_0 [f(x_n, y, z) - f(x_{n-1}, y, z)] \\ \text{and} \quad r_{n+1} &= R_0 - R_0 f(x_n, y, z); \end{aligned} \quad (15)$$

where  $R_0$  is the total resistance of the functional potentiometer, which remains constant.

The functional resistances  $r_0, b_1, b_2, \dots, b_n$  and  $r_{n+1}$  are functions only of the two variables  $y$  and  $z$  and can be represented in the form of an "electrical conoid." In the circuit shown in Figure 7, the values of the resistances  $r_0, b_1, \dots, b_n, r_{n+1}$  are developed (with the help of tracking system consisting of servomotors SD, SD2, etc. and electronic null amplifiers EU<sub>0</sub>, EU<sub>2</sub>, etc) proportional to the values of the voltages taken from the functional potentiometers  $P_{y0}, P_{y1}, \dots, P_{yn}, P_{yn+1}$ , which are the electrical representation of the functions of two variables  $y$  and  $z$ . For this purpose, the sliding contacts of the linear potentiometers  $P_{y0}, P_{y1}, \dots, P_{yn}, P_{yn+1}$  are displaced through the shaft of the variable  $y$ , while the sliding contacts of all the shunting resistances  $R_{z1}, R_{z2}, R_{z3}, \dots$ , are displaced through the shaft of the variable  $z$ . Thus, we obtain a variation of the resistances  $r_0, b_1, b_2, \dots, b_n, r_{n+1}$  as a function of  $y$  and  $z$ . The number of sections of the potentiometer  $P_x$  and of potentiometers  $P_{y0}, P_{y1}, \dots, P_{yn}, P_{yn+1}$  depends on the form of the given function and the accuracy required in its representation. Ordinarily, in practice it is sufficient to have five or six sections in each potentiometer, which provides comparatively high accuracy. The approximation errors of both potentiometer  $P_x$  and potentiometers  $P_{y0}, P_{y1}, \dots$  basically determine the error of such a device as a whole.

The devices described make it possible to represent a very broad class of functions having tabular, graphical, or analytical expressions. The restrictions on the class of functions which can be represented by these methods are imposed basically by the physical properties of the elements (potentiometers and functional resistances).

#### Design of Functional Devices

The device developed by the author for obtaining any functional dependencies consists of a linear potentiometer, the sections of which are shunted by variable resistance boxes. The potentiometer (Figure 8) consists of a closed aluminum ring 1 with a diameter of 130-150 mm covered with insulating varnish on which is wound (with the help of a special winding machine) about 8,000 turns of thin enameled constantan wire 0.05 mm in diameter. After it is wound, the ring is pressed into a textolite housing. The enamel is removed from the face surface of the ring and the sliding contact 3 of the potentiometer moves along the contact bed thus formed.

The sliding contact is displaced along the circumference of the ring on the shaft 4 with the help of a Vernier worm mechanism 5. A rough scale is installed on the shaft 4 and a fine-reading worm scale on the shaft 6, so that the total number of scale divisions is 10,000. The brushes 1-18 (with silver contacts) are installed rigidly along the edges of the contact bed of the potentiometer. These are used to connect the shunting resistances to the potentiometer.

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sections. They touch the potentiometer turns and divide it into 40 sections (only 18 of these are shown in Figure 8). The brushes are installed along the edge of the contact bed so that the sliding contact of the potentiometer can move freely along the contact surface.

Decade boxes connected to the brushes 1-18 with a multistrand cable were used as shunting resistances. The number of shunting resistances required is determined by the accuracy required of the functional device. For this purpose, a linear approximation of the given function is made and the number of sections required is determined. The values of the shunting resistances found from calculation are set up with the resistance boxes. This calculation is not difficult and can be made in 15 or 20 minutes. Tables of the values of shunting resistances can be drawn up beforehand for well-known functions.

For the most frequently used analytical or tabular functions, special sets of resistance coils are used instead of resistance boxes. These are connected to the potentiometer brushes by means of pluggable connectors. Tests of an experimental potentiometer, made by the author according to the "sine" law, showed that with 34 taps on the potentiometer, the maximum error for  $\sin x$  was 0.01-0.03 %, i.e., the values of the function were obtained in the form of an electric voltage with an accuracy of four places. This potentiometer had a resistance of about 25,000  $\Omega$  (without the shunting resistances). Its resistances with the shunting resistances was 10,000  $\Omega$ . This potentiometer was used by the author to introduce integrands into an integrating device.

The author has also developed potentiometers with controlled wire spacing and shunting resistances for computers where the functional law of voltage distribution does not have to change. These potentiometers are machine-wound linearly with a variable instead of a constant spacing, i.e., the density of the potentiometer turns varies according to a given design. The type of spacing is established in the machine by a shaped eccentric. The use of nonlinear winding permits us to reduce considerably the number of potentiometer sections with shunting resistances. Figure 9 shows a diagram of a circular sine potentiometer made from 0 to 360°, which has only two shunting resistances (for the values  $x = 70^\circ$  and  $x = 83^\circ$ ) from 0 to 90°. The winding from 0 to 70° is made according to the sine law (as is that from 70 to 83°).

Testing of the circular sine potentiometer with a ring diameter of 80 mm and a resistance of the entire ring of 12,672  $\Omega$  showed that its accuracy was 0.2-0.3% if the potentiometer taps were set fairly accurately. The method of shunting resistances proposed has been used extensively by the author for correcting production tolerances in linear potentiometers. Because of production conditions, the maximum possible accuracy in potentiometer production is 0.2-0.3%. Shunting resistances enable us to increase this to 0.02-0.03%.

Correction of a loaded potentiometer with the help of shunting resistances is also possible in potentiometric computing circuits. The loaded potentiometer is electrically profiled according to the required functional law in dependence on the value of the load resistance.

[Appended figures follow]

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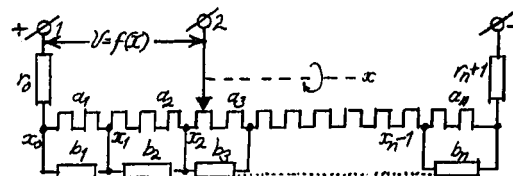


Figure 1. A Potentiometer With Shunting Resistances

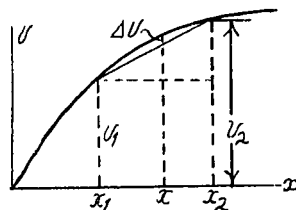


Figure 2. Determination of the Approximation Error for  $y = \text{const}$

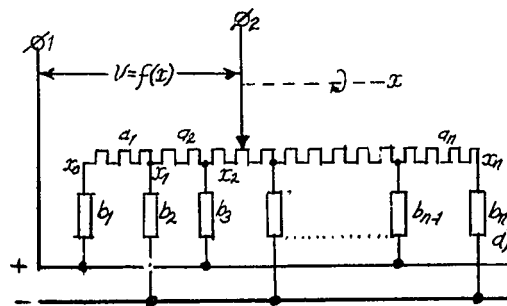


Figure 3. A Potentiometer With an Additional Resistance

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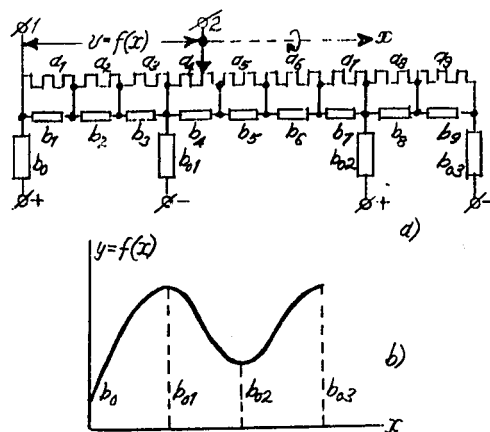


Figure 4. Representation of a Nonmonotonically Varying Function

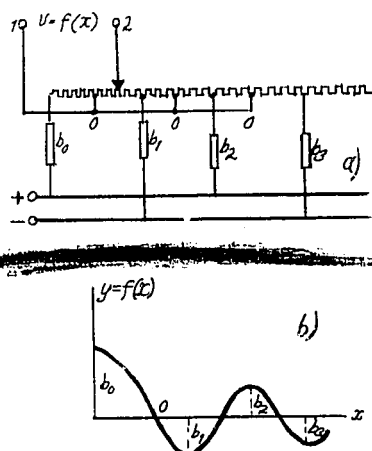


Figure 5. Representation of a Sign-Variable Function

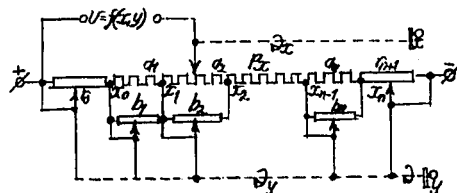


Figure 6. Representation of a Function of Two Variables

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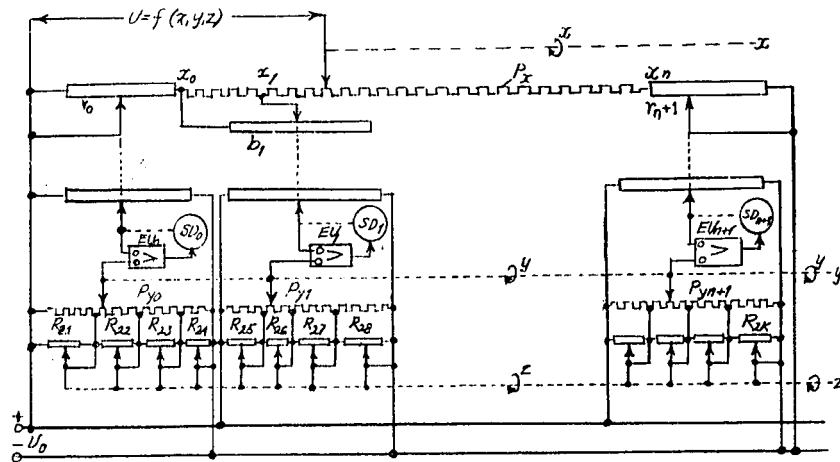


Figure 7. Representation of a Function of Three Variables

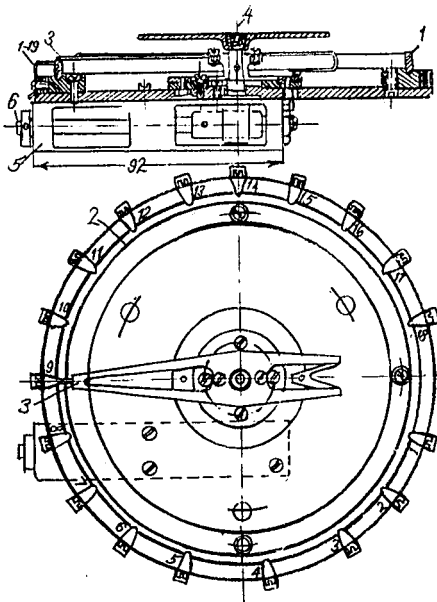


Figure 8. Potentiometer Construction

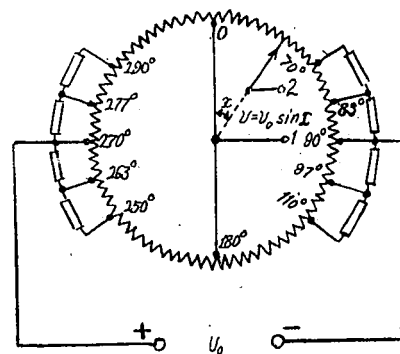


Figure 9. A Sine Potentiometer With Two Nonlinear Winding and Shunting Resistances

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